

Teräsrakenteiden maanjäristyskestävyyden FEM – analyysin sudenkuopat

Rabbit holes in seismic design of steel structures using modern design codes like EN 1998-1

RAK TEK SOLUTIONS OY

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Tekijä: DI Toomas Kaljas

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1. Response spectrum and pushover analysis

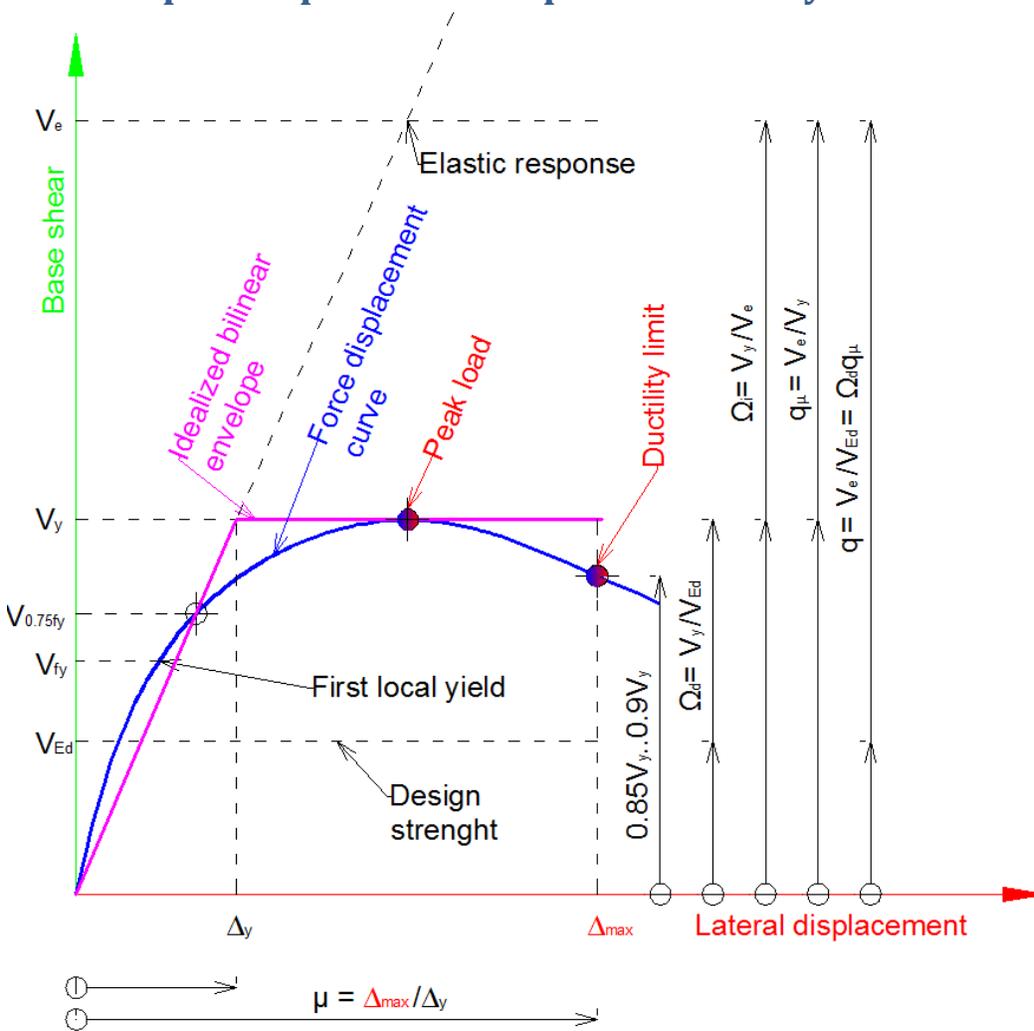


Figure 1: Relationship between strength, overstrength, ductility and behaviour factors.

V_{Ed} is load level according to design response spectrum

V_e is load level corresponding to elastic response spectrum

V_y is the maximum load on real pushover analysis

Blue curve is real horizontal force displacement curve (pushover curve)

Magenta bilinear curve represents idealization of real pushover curve.

Δ_y is the yield limiting displacement in idealized bilinear curve

$\Delta_{y_{max}}$ is the maximum acceptable displacement after which structure becomes dynamically unstable. Usually taken with small reduction of 10%..15% from maximum peak load.

Overstrength parameters:

Observed overstrength:

$$\Omega_d = \frac{V_y}{V_{Ed}} \quad (1)$$

Inherent overstrength suggested by Elnashai and Mwafy (2002)

$$\Omega_i = \frac{V_y}{V_e} \leq 1 \quad (2)$$

Observed overstrength quantifies the difference between required and actual strength while inherent overstrength relates actual strength to elastic response.

Ductility:

$$\mu = \frac{\Delta_{\max}}{\Delta_y} \quad (3)$$

is the ability of a material, component, connection or structure to undergo inelastic deformations with acceptable stiffness and strength reduction?

Maximum deformations (strains) Δ_{\max} are determined based on:

- Allowable maximum material strains
- Section curvature (maximum rotations)
- Maximum rotations in plastic hinges
- Structural displacements
- Structural instability in dynamic loadings.

Behaviour factor:

$$q = \frac{V_e}{V_{Ed}} = q_\mu \Omega_d \leq 1 \quad (4)$$

Relates design forces (spectrum forces) to elastic design forces.

Conclusions and observations:

Overstrength

In most of the modern seismic codes spectral forces are given as a cost effective design alternative to more advanced methods.

However, this simplified idea of “design forces only” can easily lead to unsafe structures.

As it can be seen from figure 1, the structure experiences equilibrium forces far beyond elastic design forces (maximum peak force)

This discrepancy is not explicitly explained in most of the seismic norms; however **EN 1998-1** gives additional rules which lead to more consistent analysis.

In **EN 1998-1** the overstrength shall be taken into account through verifications of different properties.

EN1998-1 usually takes overstrength into account by multiplying seismic effects (Internal forces caused by seismicity) with various safety and purpose built overstrength factors in a following way:

$$V_{Ed} \geq V_{EG} + 1.1\gamma_{OV}\Omega_i V_{dE} \quad (5)$$

where

$1.1\gamma_{OV}\Omega_i$ represents safety and overstrength factors arising from different causes

V_{EG} represents actions which are not caused by seismicity

V_{dE} represents actions caused by seismicity

It is therefore evident that in seismic design only spectral modal analysis is not enough.

Additionally the overstrength must be anticipated based on “experience” , “norm knowledge” or in complex cases calculated.

EN 1998-1 defines several types of overstrength factors, some of which are not called overstrength factor

Example:

For behaviour factors, basic value is given, which can be enhanced by:

$$\frac{\alpha_{ult}}{\alpha_1} \leq 1.6 \quad (6)$$

α_1 is multiplier of horizontal design seismic action at formation of first plastic hinge in the system

α_u is multiplier of horizontal seismic design action at formation of global plastic mechanism

This multiplier essentially is an overstrength factor, since it modifies behaviour factor, which in the end modifies design forces.

EN 1998-1 allows determining this factor with:

- a) Pushover analysis
- b) Adaptive pushover analysis

However, the value is limited to **1.6** even if the analysis indicates higher potential values.

Limiting the **overstrength factor** does not make sense since it is a physical measure which will tell how much the design forces will actually increase in structure. Limiting the **behaviour factor q** to 8 would make sense from conservatism point of view, but limiting effective **overstrength** to some certain predefined value is not consistent with nature.

Let us consider here a simple or more complex frame. For “overturning moments of foundation” designers need to know accurately how high forces will be generated due to seismic events. If “**overstrength enhanced**” behaviour factors are used, the design forces in foundations shall be increased due to overstrength.

“EN 4.4.2.6 The action effects for the foundation elements shall be derived on the basis of capacity design considerations accounting for the development of possible overstrength, but they need not exceed the action effects corresponding to the response of the structure under the seismic design situation inherent to the assumption of an elastic behaviour ($q = 1,0$).”

For efficient foundation design, pushover analysis must be performed and hence it is really not an option. (**q = 1?**)

Without realistic and suitable **overstrength factors** foundation are simply underdesigned or grossly conservative and non-competitive.

Similar conclusions can be made virtually for all **non-dissipative** components in the structure. Bottom line: DO NOT OVERDESIGN FUSE (**dissipative**) ELEMENTS, these are essential for overall safety

Ductility limit

Ductility limit or the maximum allowable deformation (rotation, strain etc.) are of very much importance in seismic design. For strains, ultimate value can be defined by ultimate strain. For bent members curvature limits can be established based on maximum strains.

However, when designing a new type of structure, one would be interested in its real behaviour and thus the behaviour factor.

For that purpose the maximum allowable deformation should be determined based on:

- a) Limiting deformations in seismic event (**ANNEX B EN 1998-1**)
- b) Accepting certain small reduction in the maximum load carrying capacity (strength deprecation)

While deformations shall be limited in very flexible structures (frames) the acceptable strength degradation is limiting factor in stiffer structural systems.

EN 1998-1 ANNEX B provides target deformation for certain frequency by accounting for plasticity in structural vibrations periods. However, plasticity is not accounted in mode shapes while performing pushover analysis. (adaptive pushover analysis could perform better). Alternatively one could simply analyse force displacement curve and investigate the structural performance based on that curve alone.

Due to negative slope in strength the maximum displacement must be decided based on the structural integrity and dynamic stability assessment. The question arising here is how much can the strength be safely reduced before the structure becomes dynamically unstable? The usual rate is **10%** to **15%** from maximum load value => shall be verified.

Annex B does not tell how much the strength can be reduced (degraded). Alternatively and most often conservatively the maximum load is adopted as a limiting parameter for deformations.

P-Delta (sway) and dynamic stability

EN 1998-1 and many international codes use to account structural sway effects through so called “inter-storey drift sensitivity coefficient” which is given as:

$$\theta = \frac{P_{tot} \cdot d_r}{V_{tot} \cdot h} \leq 0.1 \quad (7)$$

θ is the inter-storey drift sensitivity coefficient

P_{tot} is the total gravity load at and above the storey considered in the seismic design situation

d_r is the design inter-storey seismic drift

V_{tot} is the total seismic storey shear

h is the inter-storey height.

NOTE: The drift sensitivity coefficient is derived based on rigid ($EI = \infty$) beam-column supported by linear spring and hence gives only non-conservative upper bound value for this nonlinear eigenproblem. In cases high ductility equation 7 can be used as an upper bound value. The lower bound value, corresponding to Euler buckling is:

$$\theta = \frac{\pi^2 P_{tot} \cdot d_r}{12 V_{tot} \cdot h} \leq 0.1 \quad (8)$$

Equation (8) gives correct lower bound eigenvalue for most of structures where Euler buckling is the dominant stability mode. The equation 7 which is used in the norms is simply non - conservative and misleading; this shall be verified with FEM analysis.

The stability is considered to be significant in cases where nonlinear eigenvalue is causing error more than 10% in analysis. It is advised to use simplified magnification factor for sway effects:

$$\frac{1}{1 - \theta} \quad (9)$$

NOTE: Although in many norms it is advised to correct all the terms (internal forces) with magnification factor given in equation 9, this shall not be done. Only the displacements can be magnified with equation 9.

It is advised in **EN 1993-1** and **EN 1992-1** to amplify first order effects with amplification factor given in equation 9. Only the effects which are causing displacements used to calculate “drift sensitivity coefficient” in eq. 7 and 8 can be magnified.

However the magnification factor given in (9) does not consider the redistribution of first order internal forces due to geometric stiffness effects. These magnifications should be used with utmost care and only in cases where all members happen to have similar stiffness and axial load distribution.

If the distribution varies (stiffness/axial force) it is possible that some elements in the structure get negative stiffness while the total stiffness of the overall structure still remains positive.

This will mean that the negative stiffness elements load rest of the structure instead of supporting it (“traitor” members). It can mean that due to redistribution some of the members will become unstable in further loading. => Proper tools (compatibility, equilibrium and stress strain relations) shall be used.

Dynamic loading “inter-story drift sensitivity coefficient” in the **EN 1998-1** norm has an upper bound of **0.3**, while up to **0.2** the correction allowed to be made with simplified equation (9). Since the critical load ratio from **0.2** to **0.3** is not much, it does not matter which way the drifts are in statically magnified (simple formula or full FEM).

It does matter if some elements have negative stiffness and this is independent of so called “inter-storey drift sensitivity factor”.

Effects of sway in analysis:

- Sway reduces eigenfrequencies, increases periods and hence reduces modal loads
- Plasticity or hyperelasticity (superelasticity) further modifies effective periods (not normally accounted in linear modal spectrum analysis)
- Sway increases the risk of dynamic instability.

In a simplified and quick way the dynamic stability can be estimated by modifying the elastic stiffness factor with following:

$$\frac{\Delta_y}{\Delta_{\max}} < 1 \quad (9)$$

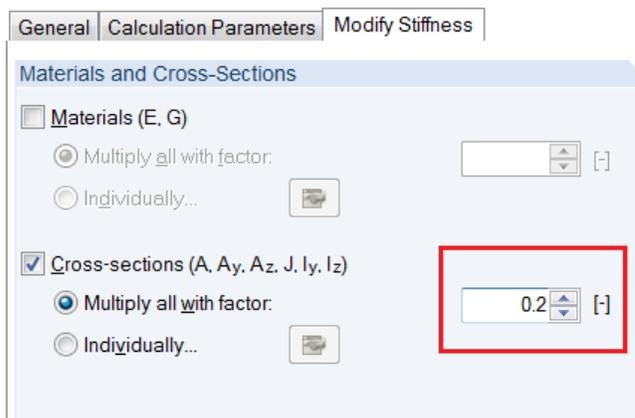


Figure 2: Modification of geometric stiffness in RFEM5. www.dlubal.com

In a more elaborate analysis the plasticity and non-linearity are explicitly accounted for in every element level.

Primary and secondary torsion

In general seismic and other design norms secondary torsion arising from non-equal biaxial loading of structures and members are not explicitly handled.

The current state of art in design method in design norms is that the sway differential equations are solved in an uncoupled fashion one plane at a time.

Example: Uncoupled governing differential equations:

$$E \cdot I_y \cdot u'''' + P \cdot u'' = q_y \quad (10)$$

$$E \cdot I_z \cdot v'''' + P \cdot v'' = q_z \quad (11)$$

$$E \cdot I_\omega \cdot \beta'''' + \left(P \cdot \frac{I_p}{A} - G \cdot J \right) \cdot \beta'' = m_x \quad (12)$$

In reality equations are coupled through torsion in multiple ways. A simple example can be as follows:

$$E \cdot I_y \cdot u'''' + P \cdot u'' = q_y \quad (13)$$

$$E \cdot I_z \cdot v'''' + P \cdot v'' - P \cdot (-z_0) \cdot \beta'' = q_z \quad (14)$$

$$E \cdot I_\omega \cdot \beta'''' + \left(P \cdot \frac{I_p}{A} - G \cdot J \right) \cdot \beta'' - P \cdot (-z_0) \cdot v'' = m_x \quad (15)$$

It is obvious from equations **14** and **15** that the torsion is coupled with bending. Similar couplings can be introduced between equations **13**, **14** and **15**. This equation set shall be solved simultaneously.

An example of secondary torsion for a cantilever member is shown in case of linear elasticity in figure 3, where IPE 300, h = 4.5m cantilever member is subjected to vertical force equal to 100kN and lateral forces of 10kN in both principal directions. The secondary torsion is significant and will cause eventual failure of the member much sooner as compared to “traditional” uncoupled “P-delta” solutions.

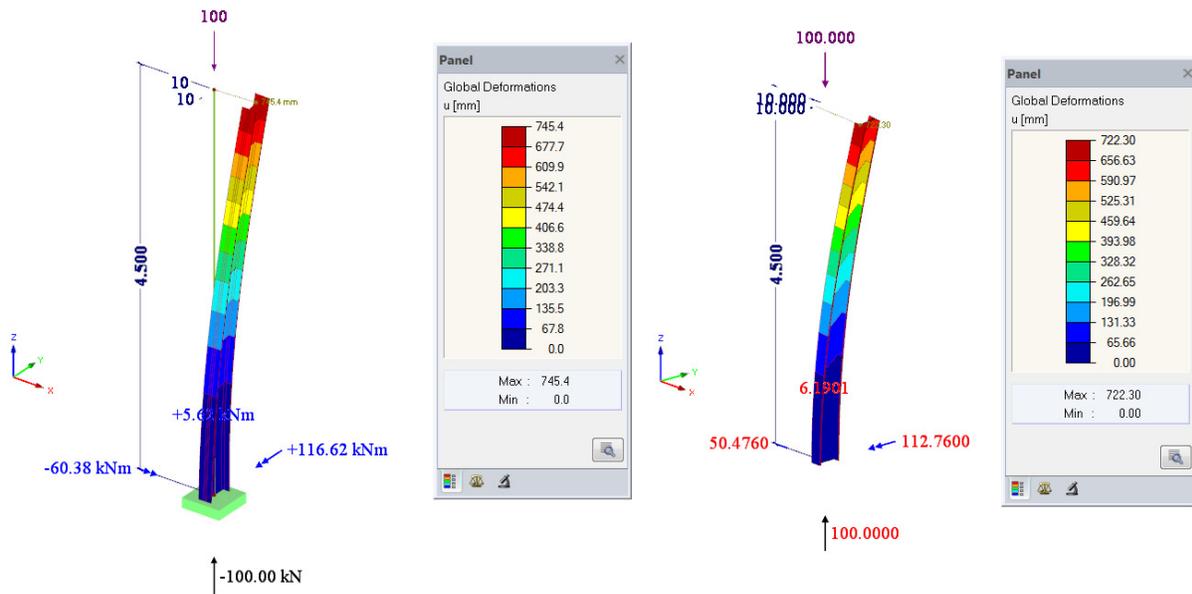


Figure 3: Secondary torsion (6 DOF free torsion = left, Vlasov torsion = right) IPE300, $h = 4.5\text{m}$, Vertical force 100kN, Lateral forces 10kN in both orthogonal directions. Secondary torsion 5.6kNm and 6.19kNm for left and right model respectfully.

The primary torsion is accounted in **EN 1998-1** and other norms fairly well. It is called accidental eccentricity. The code is also guiding engineers to avoid torsional coupled solutions. This is most likely because the complexities involved and probably far beyond the current average engineering know how and current state of art design methods.

In real structures however, it might be simply impossible to bluntly enforce symmetry. It might be impossible as well from the “purpose of building” point of view (industrial, residential etc.)

The norms should be developed to better handle equilibrium. First of all the very basic steel and concrete norms shall be elevated to new levels.

Instead of giving simple hand equations “as a solution” the norm shall state essentially important basics like:

- Compatibility
- Equilibrium
- Constitutive modelling
- etc.

And the calculations shall be left calculations for engineers. Let the market find the best ways to fulfil basic essential mechanics requirements.

The incorrect readymade solutions in norms really do not advance science and creativity, innovation, but rather help to build the union. (European Union or any other similar)

Concentric braces and simple linear static analysis (modal spectrum)

It can be often encountered in real design that concentric braces have been used to stabilize structures. However verification and design is usually done with linear analysis.

Contradiction:

1. **EN 1998-1** allows using concentric braces, but only tension members can be accounted in capacity analysis.
2. Alternatively advance pushover analysis can be performed
3. Neglecting tension members (shown in figure 4) leads to non-linear FEM analysis. How can non-linear analysis be performed with conjunction of linear spectral analysis?

Since the loads are not summed in **SRSS** or **SQS** but the internal forces, non-linear analysis is “virtually impossible”.

Perhaps non-linear analysis can be done by analysing one mode at a time in one direction and then summing the results of internal forces.

This leads to greatly increased manual interference and in modern “busy times” highly unlikely done in engineering practice.

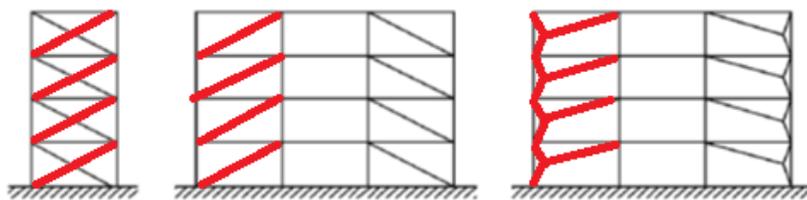


Figure 6.2: Frames with concentric diagonal bracings (dissipative zones in tension diagonals only).

Figure 4: *With concentric braces nonlinear analysis is needed!*

1. Plasticity, non-linearity and superelasticity effects on natural frequencies

In current seismic development few new trends are upcoming and with possible contradiction in final seismic safety.

Traditionally structures have been designed with the goal to achieve very stable hysteresis curves hence damping in the structure. Damping occurs when the material is rate (loading speed) dependent. Structural steel is rate dependent material.

For instance when loading is positive steel yields after yield strains have been achieved. Reversing velocity means changing the material behaviour back to elastic.

Hence the structural steel is history dependent and in some sense perfect absorber.

As an alternative to steel there are so called **self - centered post-tensioned systems** and or “**shape memory alloys**”.

Contradiction:

While superelastic structures can produce exactly same pushover curve (force displacement) as plastic materials, their behaviour under dynamic loading is considerably different.

Superelastic (hyperelastic) materials are rate independent (independent of loading speed) and hence these materials and solutions cannot exhibit damping as plastic materials do.

Superelastic materials and solutions are independent of previous loading history, the stresses can be derived based on energy potential and all the energy absorbed in spring can be fully recovered.

Example from literature:

“**Superelastic** nickel-titanium NiTi shape memory alloys (**SMA**) have also been considered by researchers for their ability to recover their original shape upon unloading after **plastic** deformations.”

It remains unclear how can “**superelastic**” material have **plastic** (unrecoverable) deformations.

It is also known from basics of mechanics that increase in flexibility increases the natural period for nonlinear and superelastic solutions. (See ANNEX B in EN 1998-1 or calculation of critical load multiplier (8))

The question to be therefore asked is how the lack of damping property affects dynamic properties of plastic and superelastic structures.

Note that the stiffness itself is directly influenced by the amplitude of lateral loading (see also adaptive pushover analysis)

Hence natural period of the structure does not have any unique value. If the loading happens to be suitable structure excitation can be violent, if not suitable the structure will become “out of tune”.

In seismic design the interest is not interested on steady state solutions, but rather transient phase.

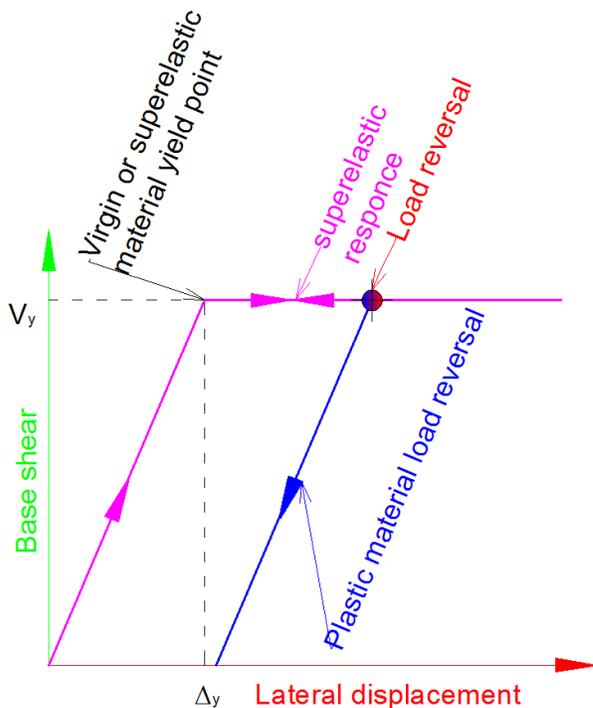


Figure 5: Response from plastic and superelastic material.

Example:

In following example a single degree vibrator which has natural period of $T_n = 7.69s$ is considered. Structure yields under deformations which exceed **0.2m**. From that the yield force is **1kN**. Structure behaves ideally and nonlinearly like shown in figure 5.

The structure is excited with its own linear elastic eigenfrequency (hence one shall expect a resonance)

The damping ratio is **0.00%** from viscous critical damping. All observed damping must therefore come from other structural properties like dissipation in plastic yielding.

Two different behaviours are investigated:

- a) Ideally elastic – plastic
- b) Superelastic

The SDOF vibrator is subjected to period of **60s** sinusoidal excitation of 5% of gravitational excitation. After **60s** the structure is followed for further **40s**. Time history analysis is therefore performed for total of **100s** time period.

Explicit time integration is used to obtain solution with time increments of **0.0001s**. Hence time is discretized into 1 million increments.

The natural frequency is **0.13Hz**, and due to **7.69s** period one should expect to see $100s/7.69s = 12.995s$ vibrations during calculated period.

Results are presented for accelerations, velocities and displacements in figures 6 and 7 for plastic and hyperplastic solutions respectively.

It can be seen from figure 6, that plastic material SDOF vibrator has following solutions:

1. Steel yielding dampens the vibration to **12** vibrations (from **12.995**)
2. Solutions has permanent deformation, around which steady state after **60s** occurs
3. Steady state from higher magnitudes to smaller ones in **60s** occurs immediately

On the other hand superelastic material solution shown in figure 7 is substantially different.

1. It has much longer natural period (60s onwards, no damping)
2. Deformations magnitudes are much higher
3. Natural period has increased

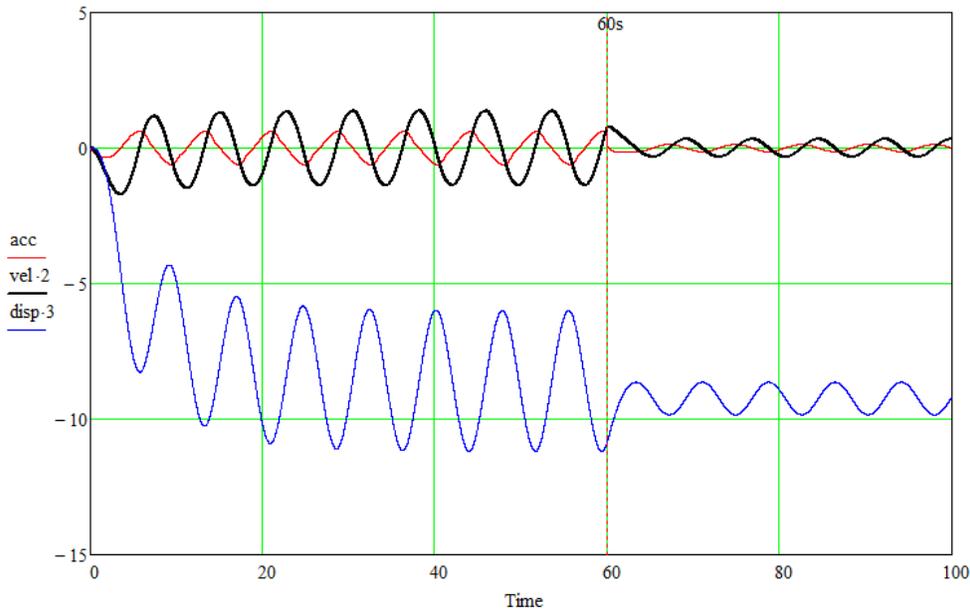


Figure 6: Accelerations, velocities and displacements of plastic solution

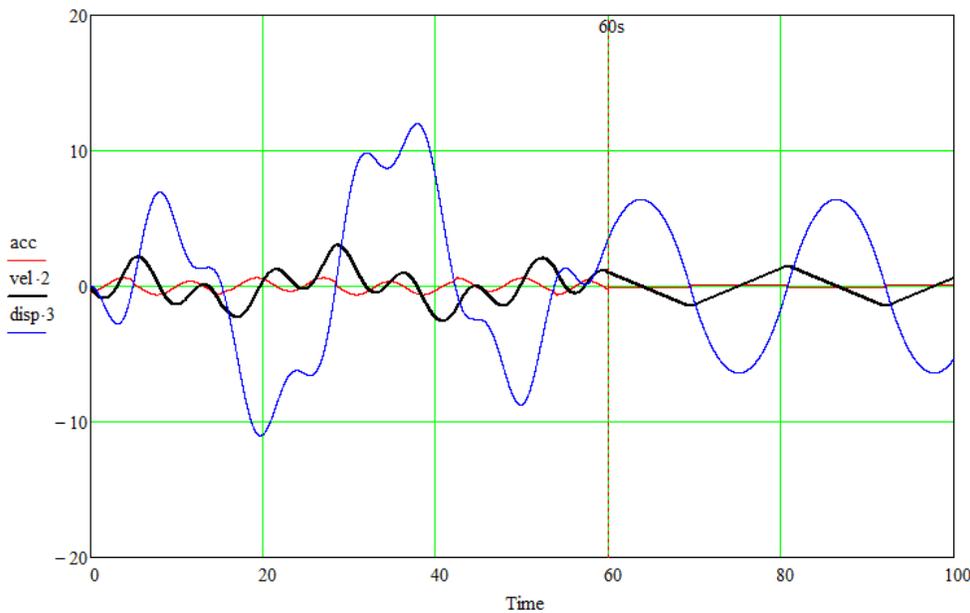


Figure 7: Accelerations, velocities and displacements of superelastic solution

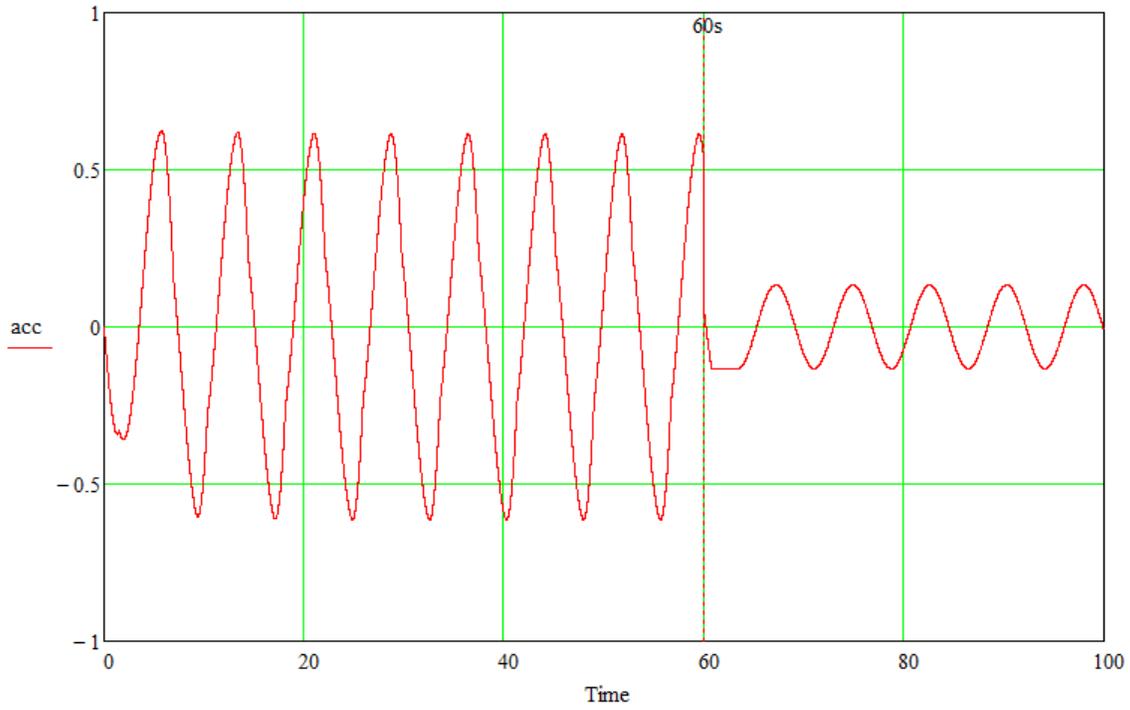


Figure 9: Accelerations of plastic structure

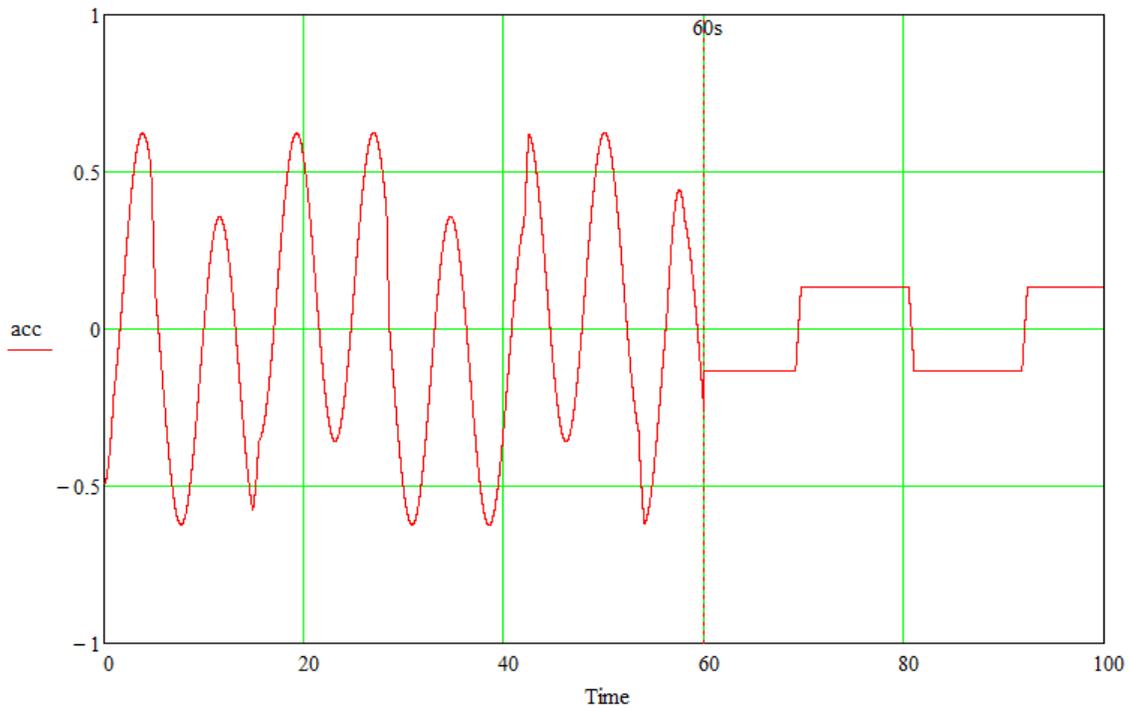


Figure 10: Accelerations of superelastic structure

It can be further observed from figure 8 and 9 that the plastic structure dampens to steady state undamped free vibration immediately after excitation ends in 60s.

Superelastic material structure however vibrates with much longer period and experiences constant accelerations.

The accelerations in both cases are limited to “yield” limit. Hence it can be concluded that the structural resistance determines the accelerations rather than its elastic stiffness.

While accelerations are limited in superelastic solutions, displacements and velocities are not due to low (here non-existent) damping.

This can mean that different strategies for seismic displacement control might have to be considered.

Also the superelastic structures can exhaust their “ductility reserves” much easily.

The appropriate excitation will cause superelastic structure to have resonance. In figure 11 the superelastic structure has been excited **2560s** with very small harmonic force

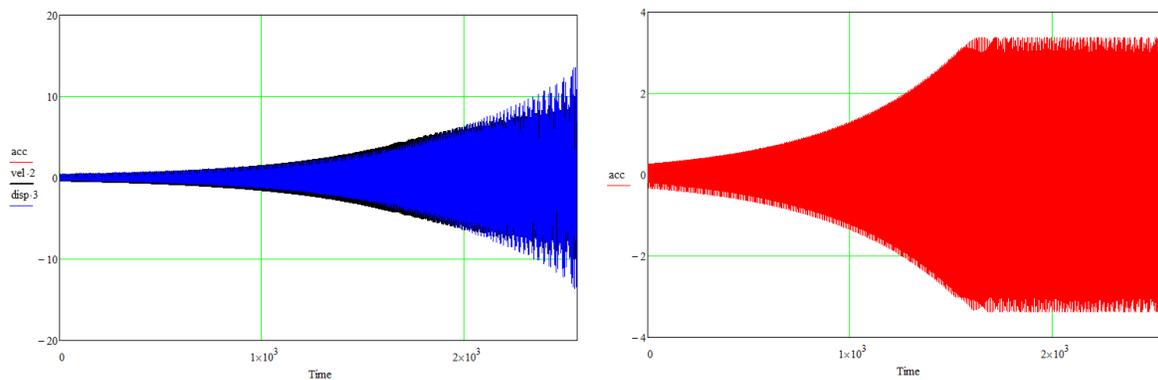


Figure 11: Displacements of superelastic structure can increase without limit (left) while accelerations are limited (right)

4. Possible Solutions and alternatives for few particular seismic design problems

Global failure mode enforcement strategy

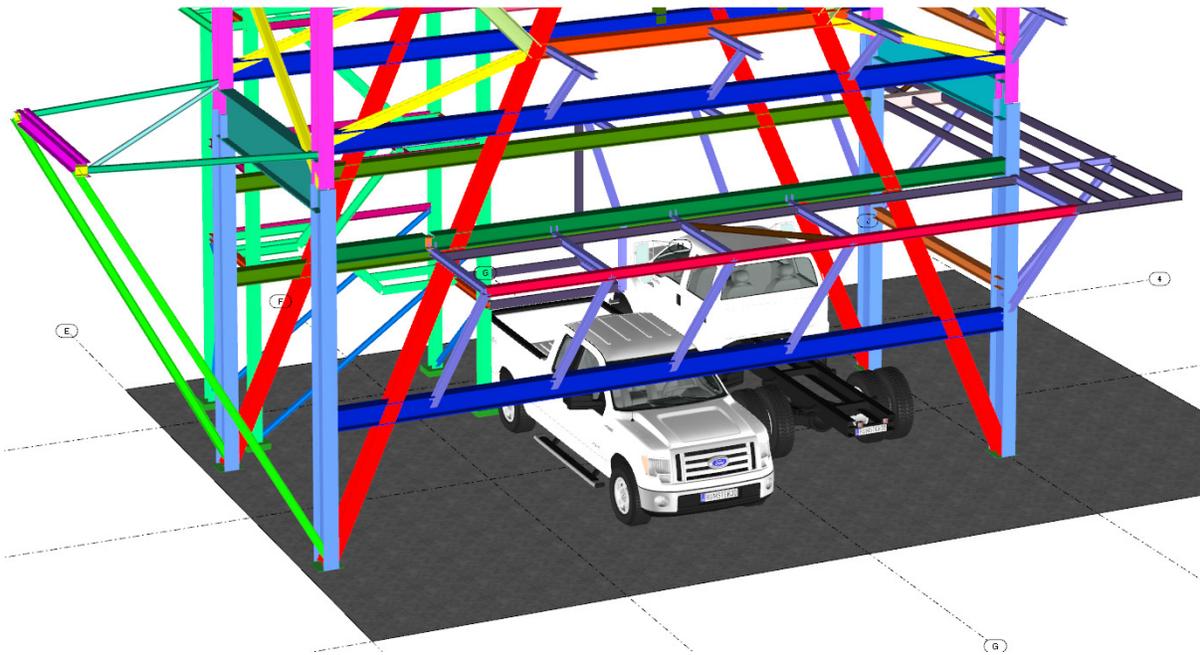


Figure 12: Industrial steel applications in *RFEM5* (www.dlubal.com)

The seismic design philosophy is to impose a ductile failure mode on the structure as a whole. In doing so part of the structure that is selected to yield in selected failure mode are detailed as high energy absorption => dissipative parts (fuse elements). Other elements shall remain elastic or plasticity shall be insignificant to their stability.

This is further enforced in seismic design codes. In order to have high ductility one must enforce global stability loss mode where all the fuses are yielding.

For that purpose seismic norm requires that the applied force/capacity ratio of individual fuse members cannot proportionally vary between floors. (Vertical regularity)

The vertical irregularity (no weak floors) cannot be avoided but in very unpractical situations only.

The seismic design norms offers little practical advice about which structural system shall be chosen and how the goal of global mechanism could be achieved except through push-over analysis.

No matter how good one makes the structural model, the non-structural parts stiffness's shall be considered in analysis model.

Example from EN 1998-1:

“4.3.1 Modelling: The model should also account for the contribution of joint regions to the deformability of the building, e.g. the end zones in beams or columns of frame type structures. Non-structural elements, which may influence the response of the primary seismic structure, should also be accounted for.”

Taking non-structural elements properly into account can be very complicated matter. However their influence can often be more significant than the designed main structure. Many steel and other lightweight structures in Finland are standing and virtually stable due to so called “stressed skin effect”, which is not usually accounted in any design calculation. Hence when the secondary structure fails the whole building fails.

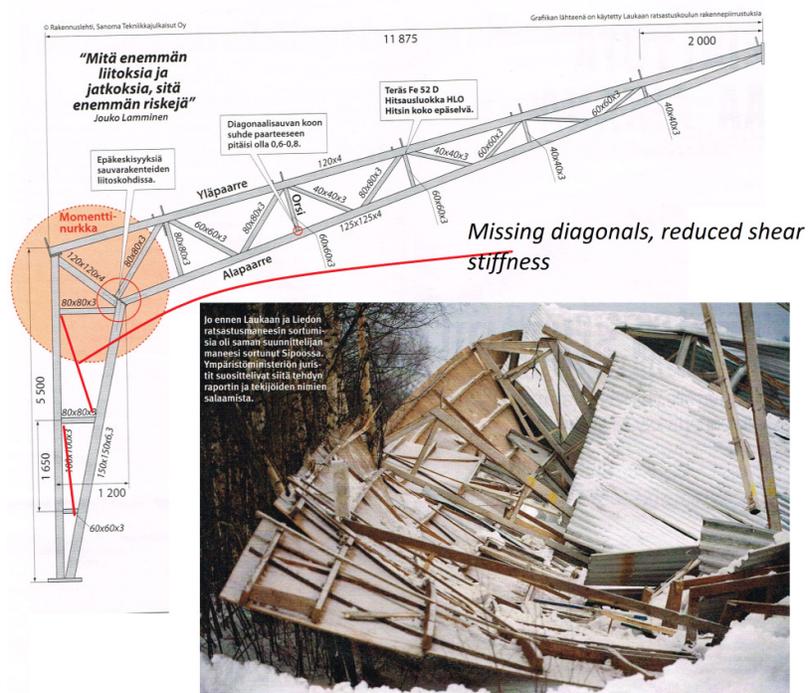


Figure 13: Failure of 3D “stressed skin effect”. Primary structures were heavily underdesigned

From figure 13 a typical structure working “due to miracle” called secondary stiffness and strength failed in Laukaa Finland on 13.02.2013.

Similar structures, working due to secondary effects, are numerous.

Due to the fact that secondary stiffness can be much higher than primary stiffness it can have very much detrimental effect on seismic performance of building or structure.

For example facades of buildings can have significant stiffness. This makes the building initially much stiffer and therefore attracts much higher seismic forces. Usually secondary stiffness like facades also have uniform strength, hence the uncontrolled secondary stiffness does influence very much detrimentally seismic performance. Lower story’s facades connection can fail leaving the upper floors intact and elastic during seismic event = > hence reducing the overall ductility significantly.

The solution could be to envision structure where failure modes happen as a single degree of freedom system. And/or where all the fuse elements are forced to yield due to redundant and efficient layout.

In 1988 Khatib et Al have proposed the zipper frame where diagonals work as fuse elements. The idea behind zipper frames was to better distribute energy between floors.

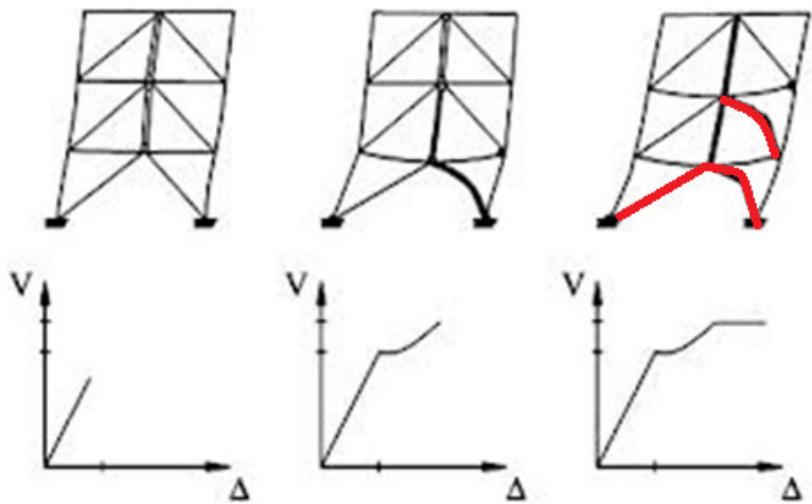


Figure 14: Zipper frames with diagonals in fuse role (reds are yielding fuses)

However Trembay and Tirca have shown in **2003** that this type of zipper frames have dynamic instability problem when subjected to severe near-field and subduction earthquake ground motions.

It is not surprising, that systems where the energy is dissipated in buckling and tension can become unstable also dynamically due to the fact that some members can have negative stiffness during analysis. (Buckling column has zero or negative stiffness depending on load level.)

However the drawback of original Trembay and Tirca can be solved by using double zipper columns and arranging the fuse elements in between the zipper columns as shown in figure **15**.

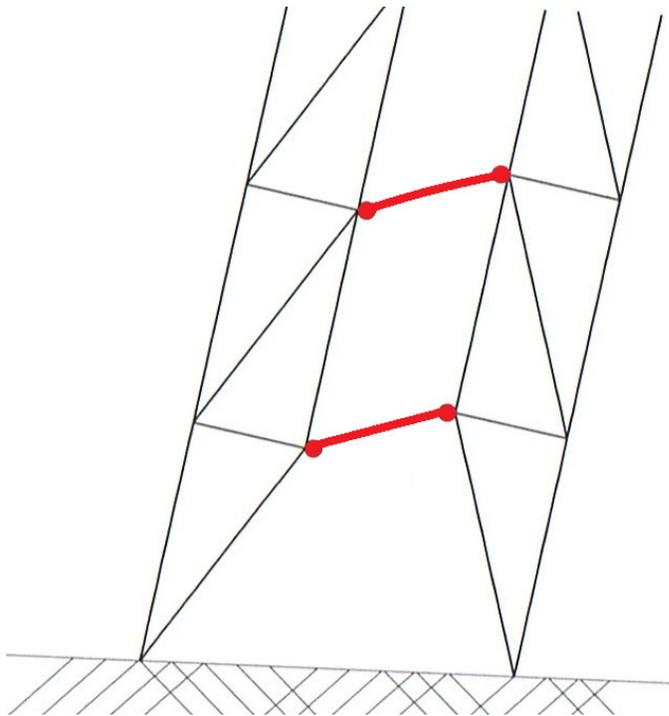


Figure 15: Zipper frames where fuse beams are used in between of double zipper columns. (**reds** are yielding fuses in between Zipper columns)

Double zipper columns in effect make the system one degree of freedom. And enforce global failure mode as a default. It also provides ample amount of stiffness as compared to **SMRF** (special moment resisting frame) and very flexible **BRB** (Bucklin Restrained Braces)

An illustrative example of **BRB** is shown in figure 16.

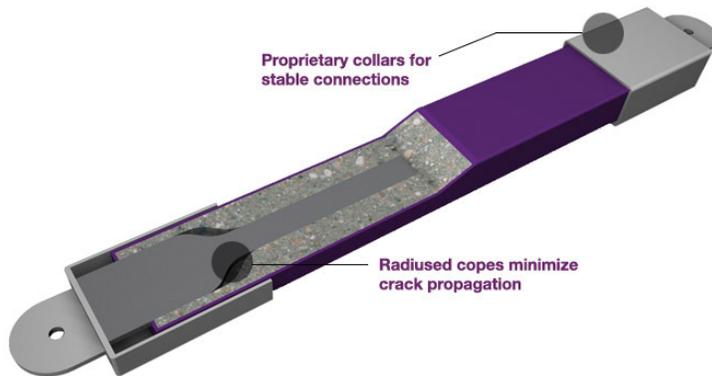


Figure 16: Bucklin restrained brace with very high behaviour factor.

Concentric diagonals and modal spectral analysis possible solutions

Due to the fact that most often concentric diagonals are used in practice and compression diagonals cannot be used in fuse roles without adequate pushover analysis, some clear yet simple analysis methods needs to be found in practice.

Previously concluded that pushover analysis is really not optional, rather a must.

However in preliminary analysis one should be able to find almost appropriate structural system. It can be done in a following manner:

1. Linear eigenfrequencies are found based on full stiffness matrix (including compression elements). This will lead to overestimation of stiffness and hence generated forces.
2. Every eigenshape shall be solved so that the base shear has clear direction, either **+** or **- x** or **y**. In solution the geometric stiffness of all seismic vertical loads shall be included
3. In analysis of each mode, the sway analysis (P-delta) shall be turned on and non-linear properties of diagonals shall be activated. Bring geometric stiffness from combinations which contains seismic only vertical forces shall be brought. Figure 17 explains how this can be done in **RFEM5**.

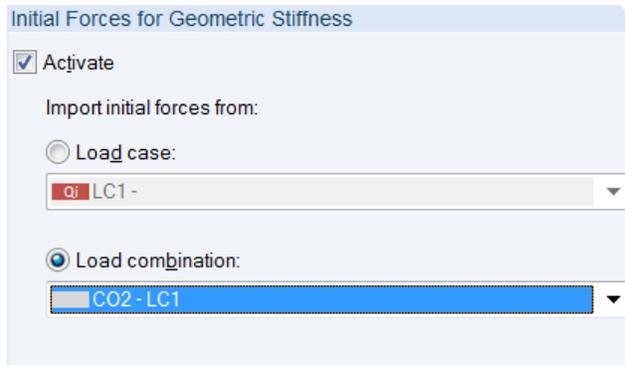


Figure 17: Use geometric stiffness from some other loading like vertical forces in seismic non-linear response analysis of individual mode in RFEM5 (www.dlubal.com)

4. The responses of each mode shall be combined
5. The diagonals (fuse elements) shall be designed to full 100% utility ratio and neglect compression forces shall be neglected which occur due to statistical combination SRSS or SQS.
6. The relevant overstrength factors for diagonals and for structure shall be estimated
7. The diagonals connections shall be designed nominally much stronger than diagonal yield limit.

NOTE! This is easier said than done. Specially manufactured casted connection parts are available on the market as shown in figure 18.



Figure 18: Casted end connections for concentric braces.

8. After completing diagonals and column design the overturning moment from each dynamic mode shall be calculated. A statistically meaningful overturning moment absolute value using either SRSS or SQS method shall be found.
9. The statistical overturning moment shall be used to calibrate some important dynamic mode as a primary loading for foundation design.
10. This meaningful mode (usually 1, but could be linear combination of multiple modes) with respect to statistical overturning moment and multiplied with all appropriate overstrength factors.
11. Nonlinear analysis for foundation design using lateral loads from point 11 and all relevant seismic forces shall be performed.
12. When using different international seismic codes, requirement of minimum base shear shall be checked. If the design loads do not fulfil minimum value, design loads shall be further increase.
13. Note! this minimum base shear is supposed to represent some way to take into account overstrength without explicitly analysing it.
14. When the structure is supported with flexible soils, the advantage of increase in period can be taken into account (SSI = soil structure interaction)

5. Literature review

- EN 1998 – 1
- Ductile Design of Steel Structures (second Editions) M. bruneau, Chia Ming Uang
Rafael Sabelli
- Seismic analysis of structures T.K.Datta
- Seismic loading on structures Tapan K.Sen
- Dynamic loading and design of structures A.J.Kappos
- Stability Analysis and Design of Structures
- Fundamentals of Earthquake Engineering Amr S. Elnashai
- Wind and Earthquake resistant buildings (Structural Analysis and Design) Bungale
S Taranath
- Designers Guide to EN 1998-1 and EN 1998-5
- Concepts and application of Finite Elements Method; Cook Malkus Plesha Witt
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- EN 1993-1
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- RFEM5 manual (<http://www.dlubal.com/>)
- RF-DYNAM manual
- Partial Differential Equations for Scientists and Engineers Stanley J Farlow.

Rak Tek Solutions Oy has set a goal to ever increasingly improve civil engineering calculations efficiency and accuracy. For that the knowledge is obtained from Aalto University and software solutions are developed in cooperation with Dlubal Ing. Software in Germany. Shall reader happen to have good and excellent ideas for further discussion and development, please do not hesitate to contact.

toomas.kaljas@rakteksolutions.fi