Frame stability following EN 1993-1-1

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Abstract

EN 1993-1-1 includes variations and possibilities to use national choices for the design of steel frames. In order to help to make the decision which variation and national parameters are to be used, it might be good to study different cases, enlarging so the examples not shown in the background documentations of the Eurocodes. In this study the parametric studies of two portal frames are shown. Some conclusions, especially for the use of the parameter α_{cr} appearing in the code, based on the results are given.

Keywords: Eurocodes, steel, frame, stability

Introduction

The novel Eurocodes for steel structures, EN 1993-1-1, include the Rankine-Merchant, or Timoshenko typed amplification factor A_f to take into account the results of so called second order linear-elastic analysis for steel frames. The second order analysis here means the elastic structural analysis, using linear stress-strain laws and applied to the geometry of the deformed structure. The amplification factor A_f is defined as (EN 1993-1-1, 2005)

$$A_f = \frac{1}{1 - \frac{1}{\alpha_{cr}}} \tag{1}$$

where

$$\alpha_{cr} = \frac{F_{cr}}{F_{Ed}} \tag{2}$$

and

F_{Ed} is the design loading on the structure and F_{cr} is the elastic critical buckling load for the global instability mode based on the initial elastic stiffness, i.e. Euler load for the frame.

The background documentation to use this kind of amplification factor can be found in many text books (e.g. Timoshenko, Gere, 1961). It is also known, that there are as many critical buckling loads, and moreover critical factors α_{cr}, as there are degrees of freedoms for the structure. For the frame there may exist infinite many degrees of freedoms. Every buckling mode represents different critical factor, and the question is which should be used in different cases? It is shown, that the use lowest positive F_{cr} means the conservative design in every case (Yang, Kuo, 1989). It is also shown, that in some cases it would be wise to look at the buckling mode before using the factor α_{cr} (Heinisuo et al, 1991). This is especially true for the frames which include trusses and similar assemblies.

It is stated in EN 1993-1-1, that the amplification factor A_f need not to be used, if $\alpha_{cr} > 10$ (in elastic analysis). The amplification factor is at the limit $A_f = 1.11$. It is stated in EN 1993-1-1, also, that the amplification factor should not be used if $\alpha_{cr} < 3$. In this case only the second order analysis is possible for the portal frames.

When considering the buckling lengths of the frames, it is stated in EN 1993-1-1 that the system lengths can be used as the buckling lengths if $\alpha_{cr} > 10$. In some applications the system lengths are used as the buckling lengths even for the range $3 \le \alpha_{cr} \le 10$.

In this study two portal frames are analyzed and designed following EN 1993-1-1. The amplification factor A_f is always present. The buckling lengths are calculated using the critical factor α_{cr} as follows, corresponding to the lowest positive buckling load,

$$\bar{\lambda} = \sqrt{\frac{f_y \cdot A}{\alpha_{cr} \cdot F_{Ed}}}$$

$$L_{cr} = \bar{\lambda} \cdot \pi \cdot i \cdot \sqrt{\frac{E}{f_y}} \qquad (3a, b)$$

where f_y is the yield strength, A is the cross-section area, $E = 210000$ MPa is the Young's modulus, L_{cr} is the buckling length and i is the radius of gyration of the column. It can be seen, that vice versa, the critical factor α_{cr} can be computed, if the buckling length of the column is known.

This means that the buckling lengths are determined using the global frame analysis. The critical factors α_{cr} are calculated using the program Robot Millenium, v. 19 and four Bernoulli-Euler typed finite elements are used for all the parts between structural joints. One case is solved using the total non-linear theory, both geometrical and material non-linearities are present in that case. That case is analysed using ABAQUS program using beam B23 finite elements.

Portal frames

Consider two plane frames shown in the Figure 1. The frames are supposed to be supported perpendicular to the frame planes at all the points, meaning the plane frame analysis can be used.

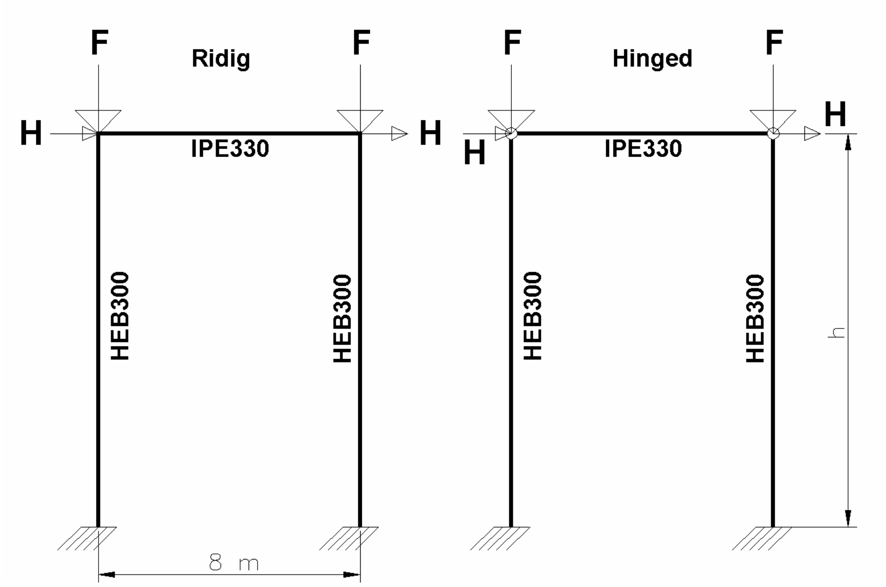


Figure 1. Portal frames considered

Both frames are rigidly jointed to the foundations. The frame called “Rigid” has rigid joints between the beam and the columns. The frame called “Hinged” has hinges at the both ends of the beam. The beam lengths are fixed to 8 meters. The heights *h* of the frames are varying between 3 to 15 meters. The beam section is IPE330 and both the column sections are HEB300. The material grade is S235 meaning the yield strength f_y = 235 MPa. The material factor γ_M is one in all the cases.

The only loads for the frames are the vertical loads *F* acting at the tops of the columns, as shown in the Figure 1. No applied horizontal loads exist. The question is what are the vertical design loads for the columns of the frames following EN 1993-1-1? Only the column design is considered in this paper. In some cases the beam design is more critical than the column design.

The frame imperfections are taken into account using imperfection loads $H = \phi F$ following EN 1993-1-1, as shown in the Figure 1.

The imperfection loads are calculated following EN 1993-1-1 for the highest frame as

$$\begin{aligned}
&\phi = \frac{\alpha_h \cdot \alpha_m}{200} \\
&\alpha_h = \frac{2}{\sqrt{h}} = \frac{2}{\sqrt{15}} = 0.51 \; and \; \frac{2}{3} \le \alpha_h \le 1 \Rightarrow \alpha_h = 0.67 \\
&\alpha_m = \sqrt{0.5 \cdot \left(1 + \frac{1}{m}\right)} = \sqrt{0.5 \cdot \left(1 + \frac{1}{2}\right)} = 0.87 \\
&\Rightarrow \phi = \frac{0.67 \cdot 0.87}{200} = 0.0029 = \frac{1}{345}
\end{aligned} \qquad (4)$$

Similar calculations hold for other frames. The results are given in the Table 1.

Table 1. Imperfection factors for frames

h (m)	15	12	9	6	4.5	3
ϕ	$\frac{1}{345}$	$\frac{1}{345}$	$\frac{1}{345}$	$\frac{1}{280}$	$\frac{1}{245}$	$\frac{1}{230}$

Now, when the load F is given, the critical factor α_{cr} can be calculated. After that we get the amplification factors A_f by which we multiply the horizontal loads ϕF, if needed. Then we define the bending moments (M_{Ed}), axial forces (N_{Ed}) and shear forces (V_{Ed}) for all parts of the frames using the linear plane frame theory.

The interaction for the axial load and for the bending moment using the Method 2 of EN 1993-1-1 is defined by checking the equation

$$\frac{N_{Ed}}{\chi \cdot N_{Rd}} + k \cdot \frac{C_m \cdot M_{Ed}}{M_{Rd}} \le 1 \qquad (5)$$

The shear loads are very low for the columns and they are not critical in this case. Also, the deflection limits are not considered in this case.

In this case we use the elastic design for the cross-sections, too, (W_e is the elastic bending modulus of the column, and the material factor γ_M is set to one) and then

$$\begin{aligned} N_{Rd} &= \frac{f_y \cdot A}{\gamma_M} = f_y \cdot A \\ M_{Rd} &= \frac{f_y \cdot W_e}{\gamma_M} = f_y \cdot W_e \end{aligned} \qquad (6a, b)$$

The buckling factor χ is calculated using the imperfection factor $\alpha = 0.34$ for hot rolled sections

$$\begin{aligned} \chi &= \frac{1}{\varphi + \sqrt{\varphi^2 - \bar{\lambda}^2}} \quad but\ \chi \le 1 \\ \varphi &= 0.5 \cdot [\ 1 + \alpha \cdot (\bar{\lambda} - 0.2) + \bar{\lambda}^2\ ] \end{aligned} \qquad (7a, b)$$

The factor k is calculated using the equations

$$n = \frac{N_{Ed}}{\chi \cdot N_{Rd}} \qquad (8a, b)$$
$$k = 1 + (\overline{\lambda} - 0.2) \cdot n \leq 1 + 0.8 \cdot n$$

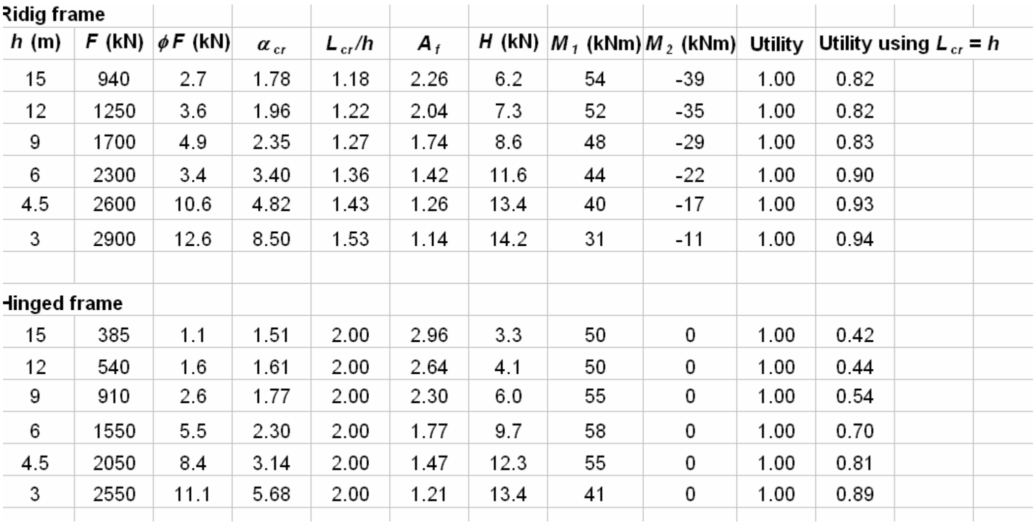
The factor C_m is for the linear bending moment distribution

$$C_m = 0.6 + 0.4 \cdot \psi \geq 0.4 \; but \text{ min } 0.9 \;\; for \; sway \;\; frames \qquad (9a, b)$$
$$-1 \leq \psi = \frac{M_2}{M_1} \leq 1$$

where M_1 and M_2 are the bending moments at the ends of the column.

After some iteration, the results shown in the Table 2 can be got. The table shows also the utility ratios if the system lengths are used for the buckling lengths.

Table 2. Results for the frame designs



Ridig frame										
h (m)	F (kN)	ϕF (kN)	α_{cr}	L_{cr}/h	A_f	H (kN)	M_1 (kNm)	M_2 (kNm)	Utility	Utility using $L_{cr} = h$
15	940	2.7	1.78	1.18	2.26	6.2	54	-39	1.00	0.82
12	1250	3.6	1.96	1.22	2.04	7.3	52	-35	1.00	0.82
9	1700	4.9	2.35	1.27	1.74	8.6	48	-29	1.00	0.83
6	2300	3.4	3.40	1.36	1.42	11.6	44	-22	1.00	0.90
4.5	2600	10.6	4.82	1.43	1.26	13.4	40	-17	1.00	0.93
3	2900	12.6	8.50	1.53	1.14	14.2	31	-11	1.00	0.94
Hinged frame										
15	385	1.1	1.51	2.00	2.96	3.3	50	0	1.00	0.42
12	540	1.6	1.61	2.00	2.64	4.1	50	0	1.00	0.44
9	910	2.6	1.77	2.00	2.30	6.0	55	0	1.00	0.54
6	1550	5.5	2.30	2.00	1.77	9.7	58	0	1.00	0.70
4.5	2050	8.4	3.14	2.00	1.47	12.3	55	0	1.00	0.81
3	2550	11.1	5.68	2.00	1.21	13.4	41	0	1.00	0.89

Analysis and discussion of the results

When considering the use of system lengths as buckling lengths for the frames considered, it can be seen, that the limit α_{cr} = 3 means for the rigid frame about 10% error on the unsafe side and for the hinged frame about 20% error on the unsafe side, if the use of buckling lengths based on the theory of elasticity is considered as a correct value. The limit for α_{cr} = 4 was in the ENV version of the standard. If the same error, say below 10% is allowed at the utility ratios, then the limit α_{cr} = 5 should be more correct in the cases considered in this paper.

It is proposed in the Finnish NA for EN 1993-1-1, that the elastic theory should be used when determining the buckling lengths for frames. During previous years, when national codes have been applied, the designers have been used to apply this rule without difficulties, so there seems to be no reason to develop this kind of simplified rule to use the systems lengths here, as proposed in EN 1993-1-1.

In many codes, e.g. those of AISC and SNIP, are given the upper limits for the slenderness of the members, $\lambda = L_{cr} / i$. In EN 1993-1-1 is given the lower bound for $\alpha_{cr} = 3$. The slenderness λ can be solved from the equations (3) above and using the notation

$$F_{Ed} = A \cdot \sigma_{com} \qquad (10)$$

where σ_{com} is the compressive stress of the column, the following results is got

$$\lambda = \frac{L_{cr}}{i} = \pi \cdot \sqrt{\frac{E}{\alpha_{cr} \cdot \sigma_{com}}} \qquad (11)$$

The slenderness as a function of α_{cr} is shown in the Figure 2 for different stress levels.

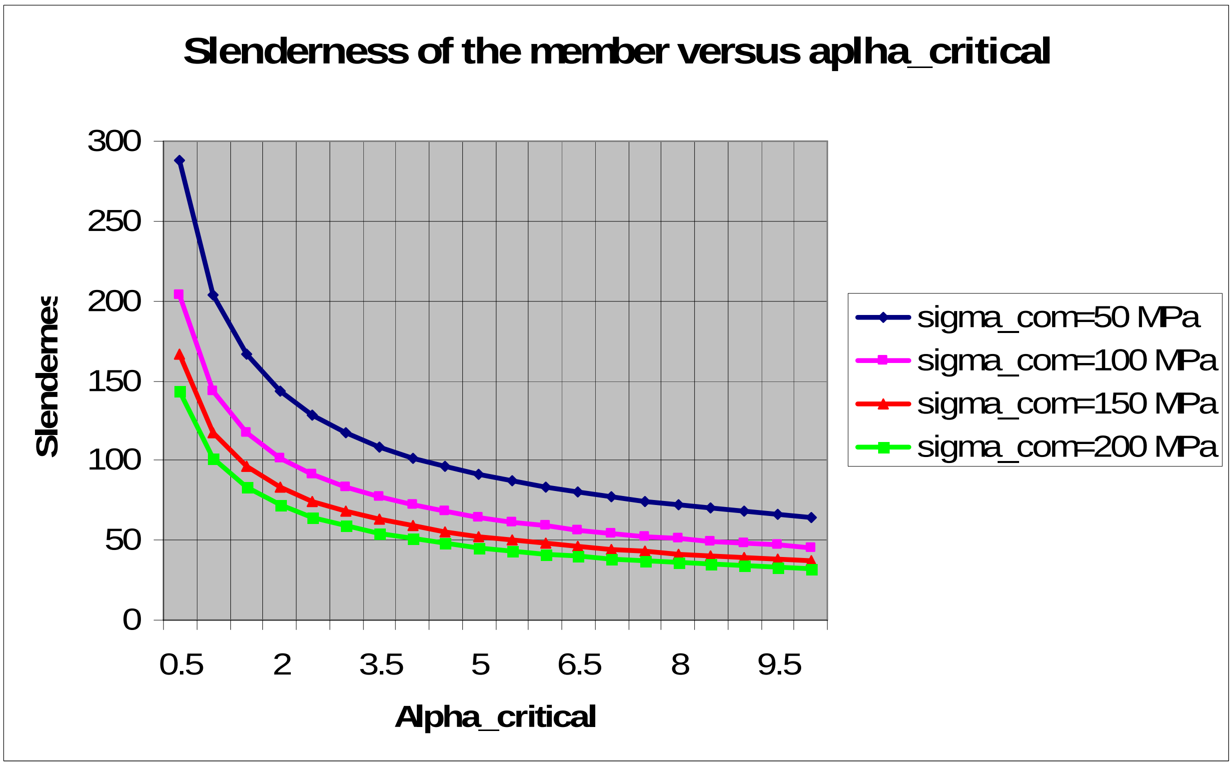


Figure 2. Slenderness of the member versus α_{cr}

It can be seen, that the limit $\alpha_{cr} = 3$ means slenderness 58, …, 117 for the cases shown in the figure. It is known, that there exist many fames in the real projects, where the slenderness is above these values. So, direct rules for the slenderness of the members are not given in EN 1993-1-1. One rule may be derived from the rule, when the bow imperfection should be taken into account in the second order theory (see Eg. 5.8 of EN 1993-1-1). Typically this means a very slender member. Using the equations given above this rule imply the following

$$
\begin{aligned}
&\sigma_{com} > \frac{\sigma_{Euler}}{4} \text{ where} \\
&\sigma_{Euter} = \frac{\pi^2 \cdot E \cdot I}{h^2 \cdot A} \text{ or} \\
&\lambda_h = \frac{h}{i} > \frac{\pi}{2} \cdot \sqrt{\frac{E}{\sigma_{com}}}
\end{aligned}
\tag{11}
$$

where h is the height of the frame as shown in the Figure 1 and I is the inertia moment of the column. In our cases the three rigid frames with h = 15, 12 and 9 m fall to this category.

When the value α_{cr} is very low, then the frame is very slender, and in many cases the horizontal deflection limits control the design. Moreover, to see what happens below the lower limit α_{cr} = 3 one case was solved using both geometrical and material non-linear analysis. The highest rigid frame (h = 15 m) was chosen as the case. The same case has been analysed in (Renkonen, 1995) applying rules of ENV.

In this case both the frame imperfection and the bow imperfection should be used following EN 1993-1-1. Two ways to give the imperfections were considered, as shown in the Figure 3. The first way is “engineering solution” and the second case is the combination of the eigenmodes, first and third. The lowest frame eigenmode (43 = 15000/345) is combined to the lowest bow eigenmode (75 = 15000/200), as shown.

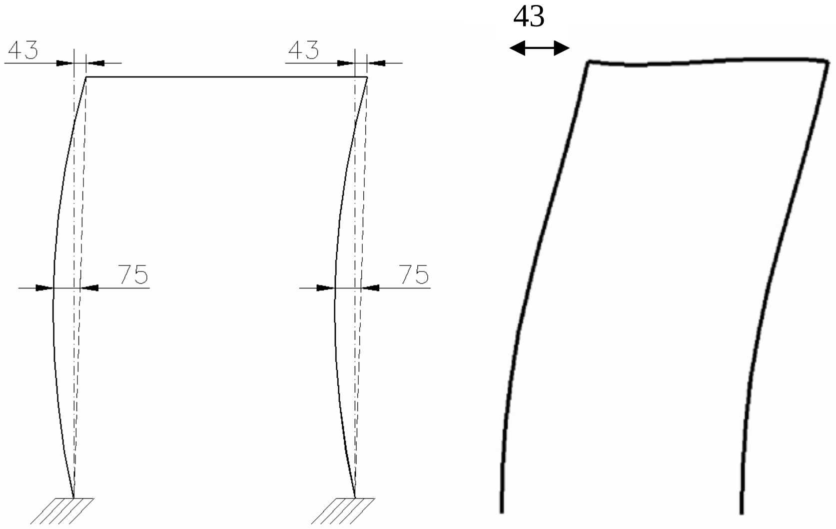


Figure 3. Imperfections for the non-linear case (not in scale)

The loads in this analysis were only two vertical loads at the corners of the frame. The material model was elastic-rigid plastic with E = 210000 MPa and f_y = 235 MPa. The program Abaqus and the beam elements B23 (ten for every member) were used. The results are shown in the Figure 4.

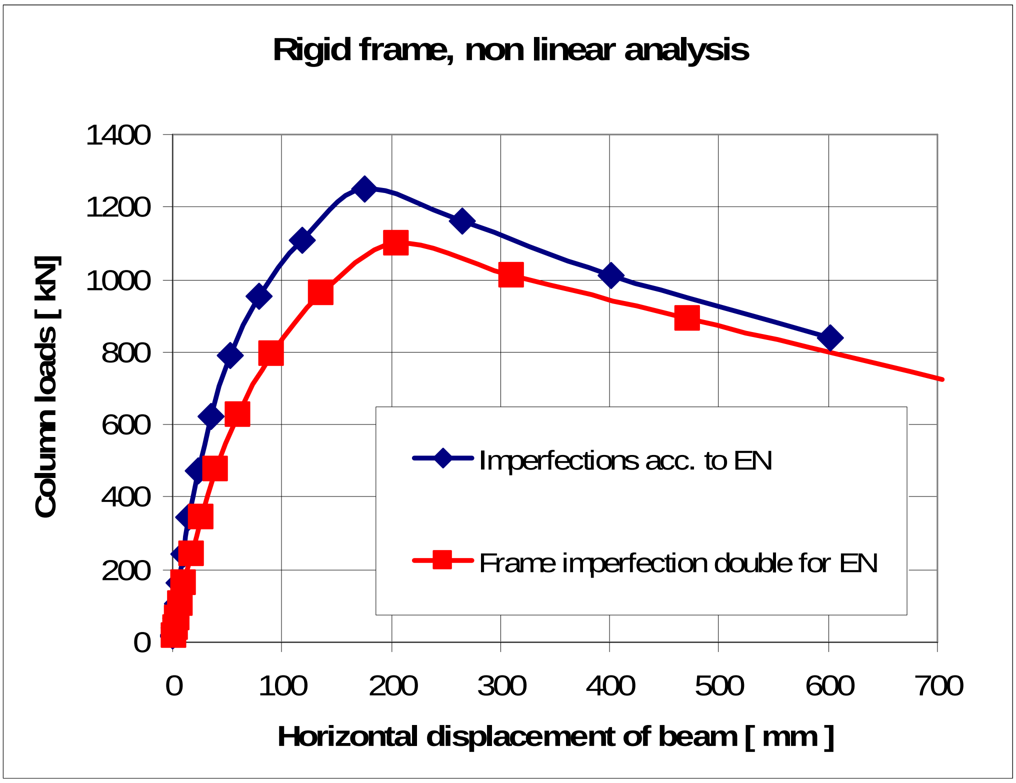


Figure 4. Rigid frame (h = 15 m), non-linear analysis results

It can be seen, that EN 1993-1-1 gives safe results for this case, because the maximum load is 1250 kN (940 kN according to EN 1993-1-1). It can be seen, that if the frame imperfection is doubled, then the maximum load is 1110 kN. Both the shapes for the imperfections presented in the Figure 3 gave practically the same results. It can be seen, also, that this frame behaves strongly non-linearly before the ultimate load of EN 1993-1-1.

When considering the automatic design system as proposed in e.g. (Heinisuo at al, 1991), then it would be nice to get rid of the limits as proposed in EN 1993-1-1 for α_{cr}. When looking at the results shown above, it may be possible to apply the design rules for frames of EN 1993-1-1 as follows

- Use the frame imperfections, or more precisely, equivalent horizontal loads, as proposed in EN 1993-1-1, bow imperfections are included in the buckling curves used in the design
- Use the amplification factor in every case, multiply all the horizontal loads by the factor
- Define the buckling lengths of members using the theory of elasticity in every case
- Use no limits for α_{cr}.

Conclusions

As conclusions of the results shown in this paper:

- The lower limit $\alpha_{cr} = 3$ given in EN 1993-1-1 seems to give different safety margins for the cases considered. Better limit seems to be $\alpha_{cr} = 5$, when all the rules appearing in EN 1993-1-1 are used.
- The use of elastic theory to define the buckling lengths in very case, as proposed in the Finnish NA for EN 1993-1-1, seems to be proper for the cases considered in this paper.
- EN 1993-1-1 gives no limits for the slenderness of the members, except for portal frames. This limit seems to be too high for the cases considered.
- In automatic design perhaps no limits are needed for α_{cr}, and apply the rules given in the text.

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